Inference and hypothesis testing

EDS 222

Tamma Carleton
Fall 2023

Announcements/check-in

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- Change in OH this week (*today* 3:15-4:15pm, Pine Room)
- Assignment 4 posted this week, likely due 12/01 but will depend on...
- ...the next few weeks. We might need 2.5 weeks for inference + time series. We will focus on going slow enough to fit it all in (we have slack time built in)

Today

Remember week 1? ...why are we we here?

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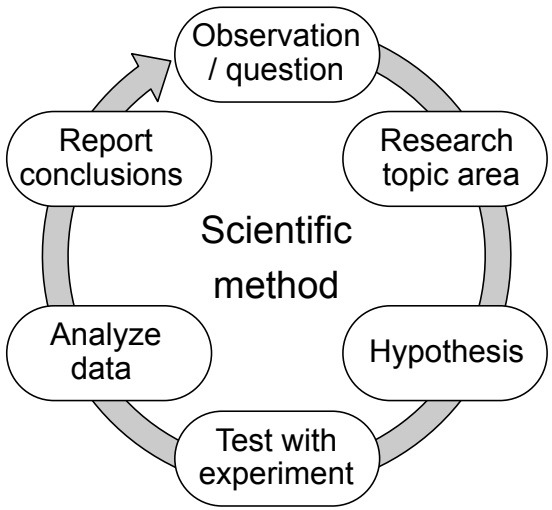
Statistics:

The science of **collecting**, **manipulating**, and **analyzing** empirical data

Statistics enables us to use environmental data to follow the **scientific method**



Today

Thinking about uncertainty

Sampling distributions

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Hypothesis testing: conceptual foundations

Null hypotheses, alternative hypotheses, rejecting the null

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Confidence

Confidence intervals

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- We have a *randomly drawn sample* and are trying to learn about the population from our sample
- But our sample statistics would have been different had we randomly drawn a different set of observations!
- This is **natural variability** and it means that all our sample statistics are uncertain estimates of population parameters, even if they are unbiased (e.g., no convenience sampling, no systematic non-response, etc.)

Why does uncertainty matter?

Key question: Is our estimate indicating anything more than sampling variability or "noise"?

- This is the question **statistical inference** and **hypothesis testing** are trying to answer

Why does uncertainty matter?

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- Does this mean there is a gender wage gap, or did we just *happen* to get a few high-earning men and a few low-earning women in this group?

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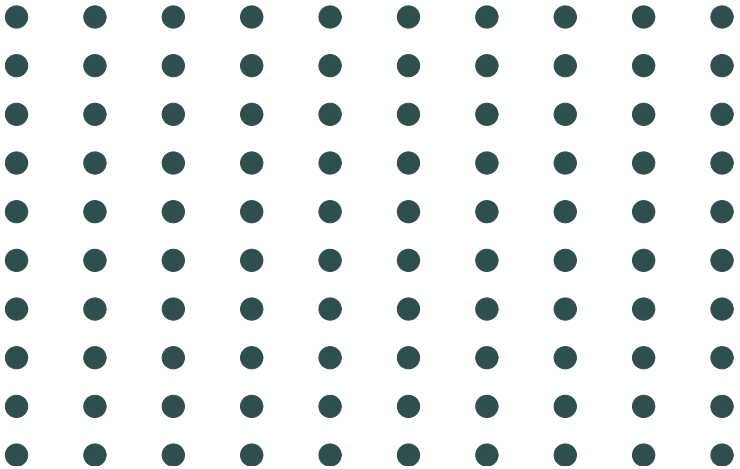
- Does this mean there is a gender wage gap, or did we just *happen* to get a few high-earning men and a few low-earning women in this group?
- If we collected another independent sample of 100, would the gap be the same?

Population *vs.* sample



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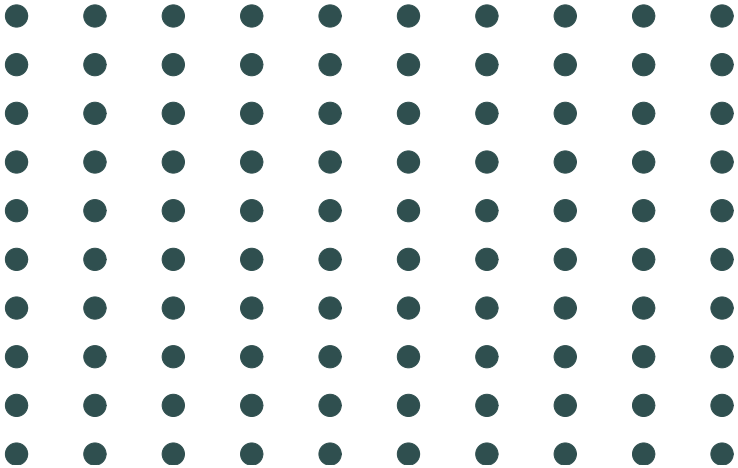

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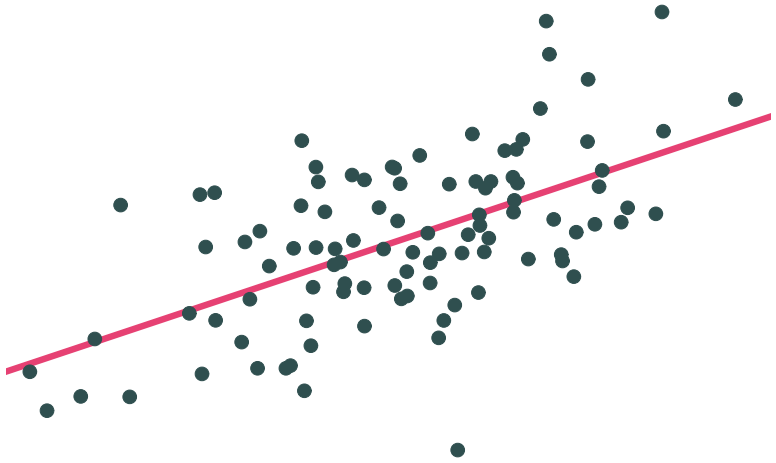
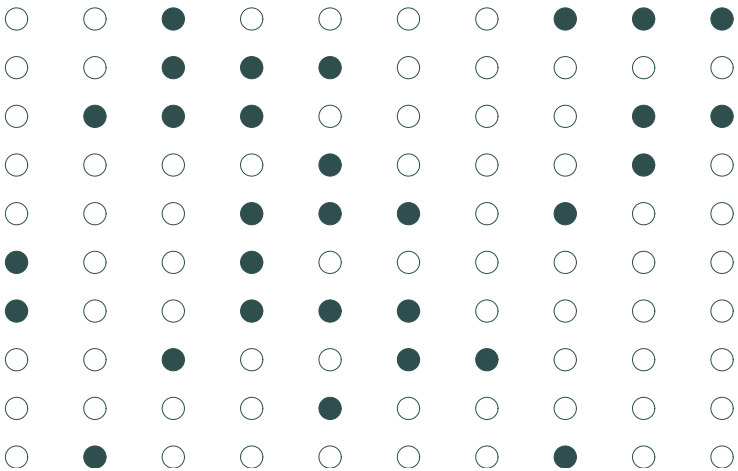


Population relationship

$$y_i = 2.53 + 0.57 x_i + u_i$$

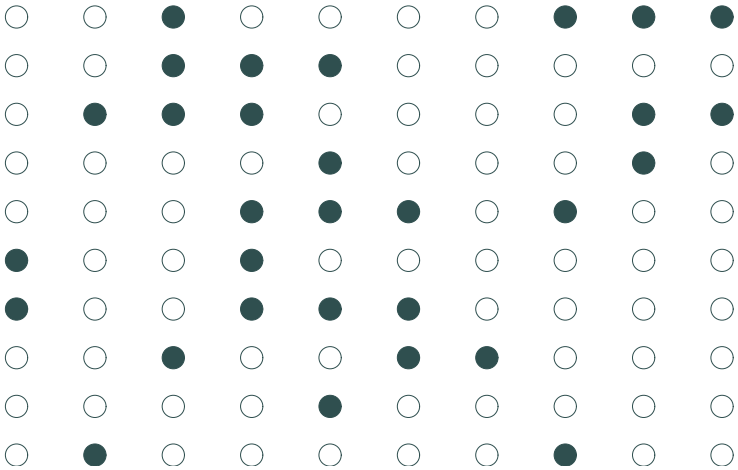
$$y_i = \beta_0 + \beta_1 x_i + u_i$$

Population *vs.* sample



Sample 1: 30 random individuals

Population *vs.* sample



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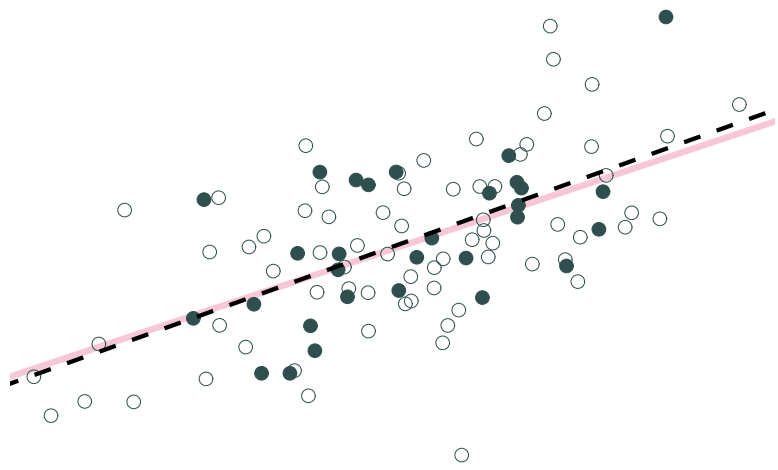


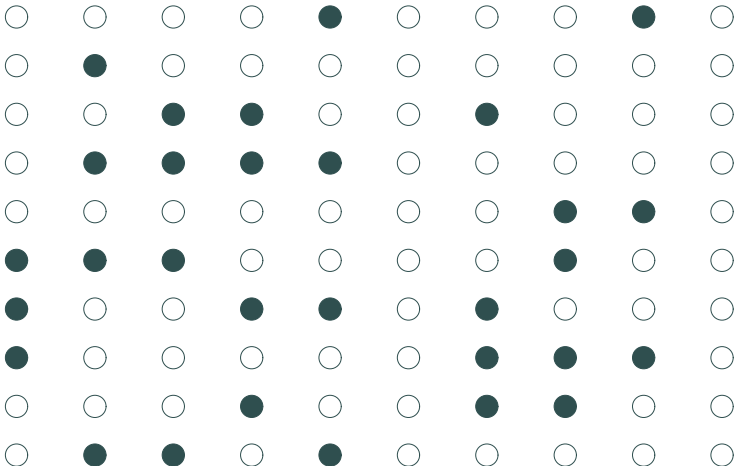
Population relationship

$$y_i = 2.53 + 0.57x_i + u_i$$

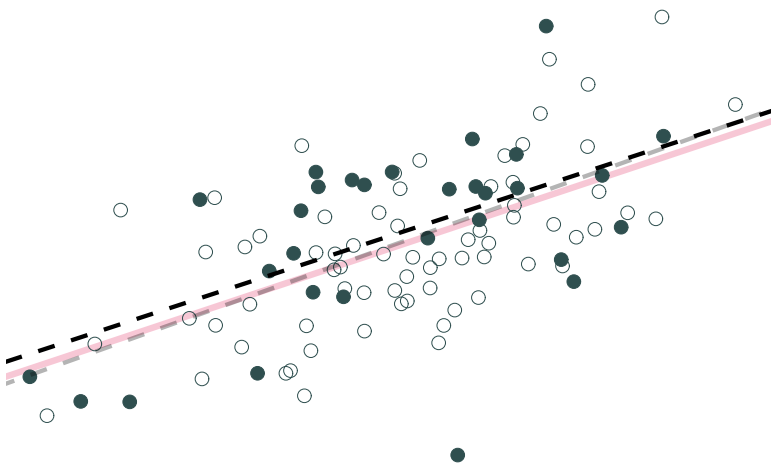
Sample relationship

$$\hat{y}_i = 2.36 + 0.61x_i$$

Population *vs.* sample



Sample 2: 30 random individuals



Population relationship

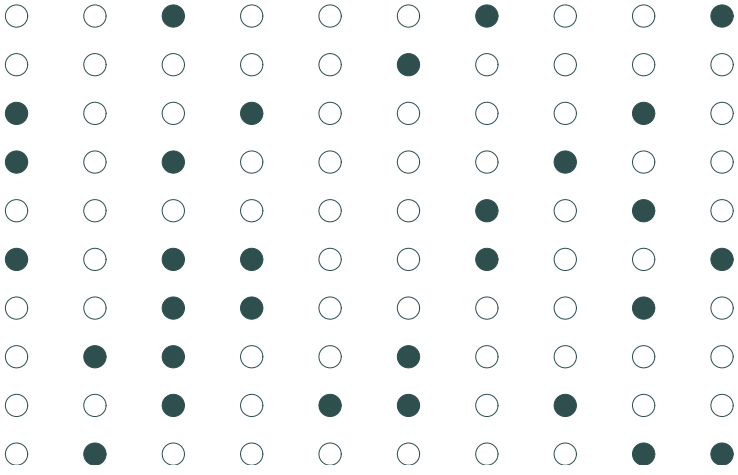
$$y_i = 2.53 + 0.57x_i + u_i$$

Sample relationship

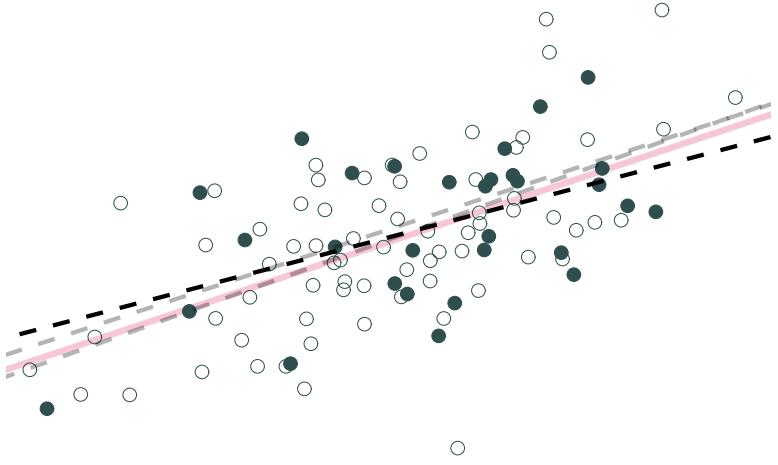
$$\hat{y}_i = 2.79 + 0.56x_i$$

Population *vs.* sample

count: false



Sample 3: 30 random individuals



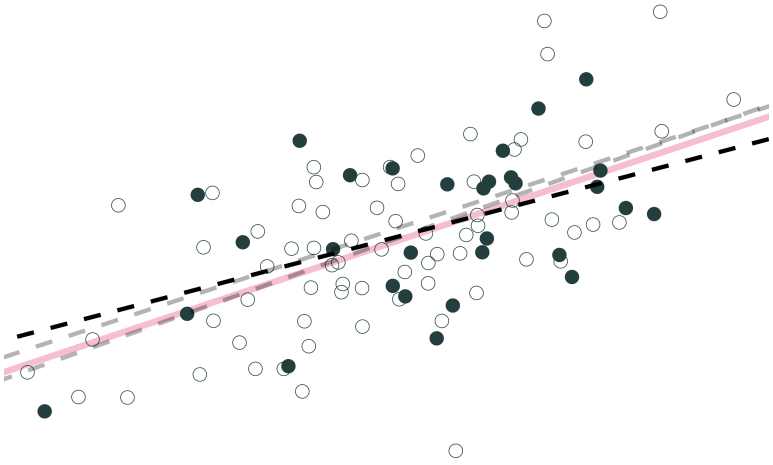
Population relationship

$$y_i = 2.53 + 0.57x_i + u_i$$

Sample relationship

$$\hat{y}_i = 3.21 + 0.45x_i$$

Population *vs.* sample



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- However, **individual lines** (samples) can really miss the mark.
- Differences between individual samples and the population lead to **uncertainty** for the statistican

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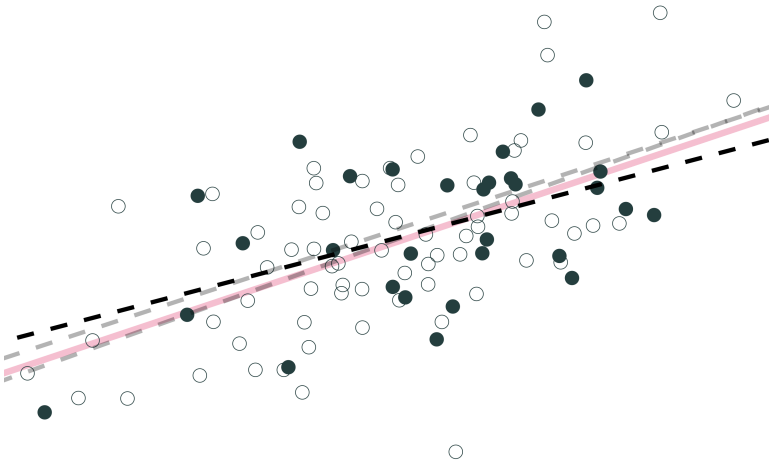

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> Keeping track of uncertainty allows us to test hypotheses about the population using just our sample

Hypothesis testing: conceptual foundations

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- H_0: **Null hypothesis** A default hypothesis that the measured quantity (e.g., sample mean, difference in means, regression parameters) is zero. In other words: whatever I recovered in my sample is due to random chance.

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- H_A: **Alternative hypothesis** A hypothesis that the measured quantity is nonzero. In other words: whatever I recovered in my sample is due to true population differences or effects.

Hypothesis testing: Example

Example: Are ducks moving north?

There are reports that Midwestern duck populations are moving northwards in response to climate change.

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You have a random sample of 100 tagged ducks in Minnesota from 2010 and 2023.

- Mean 2010 latitude: 44.27 degrees N
- Mean 2023 latitude: 46.12 degrees N
- Standard deviation 2023 latitude: 0.92 degrees

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You have a random sample of 100 tagged ducks in Minnesota from 2010 and 2023.

- Mean 2010 latitude: 44.27 degrees N
- Mean 2023 latitude: 46.12 degrees N
- Standard deviation 2023 latitude: 0.92 degrees

- H_0: The average latitude was the same in the two years. That is, $\mu_{2023} - \mu_{2010} = 0$

- H_A: The average latitude was not the same in the two years. That is, $\mu_{2023} - \mu_{2010} \neq 0$

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- 46.12 - 44.27 = 1.85

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Here, the difference in mean latitude between the 2023 sample and the 2010 sample is:

- 46.12 - 44.27 = 1.85

> Hypothesis test asks if this point estimate is actually different from zero once we account for sampling variability

Hypothesis testing: Rejecting the null

How do we choose between H_0 and H_A?

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Rejecting the null involves both a **point estimate** and a measure of **uncertainty** or spread in your data

Hypothesis testing: in practice

Hypothesis testing in five steps

The general framework for implementing a hypothesis test is:

1. **Define the null** and alternative hypotheses

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5. Based on #4, either **reject** or **fail to reject** the null

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We already know all about these two steps.

- Null and alternative hypotheses will depend entirely on the statistical question of interest.
- Data collection and point estimates (e.g., means, regression parameters, variances, etc.) we have studied at length in this class

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Let's turn to some definitions.

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A **sampling distribution** is the distribution of all possible values of a sample statistic from samples of a given size from a given population.

- The sampling distribution describes how sample statistics (e.g., mean, regression parameters) vary from one sample (or study) to the next
- This is *not* the same as the **data distribution**!
    - Distribution of your data = distribution within one sample (e.g., gives you *one* sample mean)
    - Sampling distribution = distribution across samples (e.g., gives you *many* sample means)

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For example, recall our regression above, where the population model is:

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- A regression using one sample gives us *one* set of coefficients, called the **point estimates**. For example, $\hat{\beta}_0 = 2.36$ and $\hat{\beta}_1 = 0.61$
- If we could collect 1000 samples and run that regression 1,000 times, we would recover the **sampling distribution** for each coefficient

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But how do we obtain one of these?

You only have one sample of data! Where does the sampling distribution come from?

We derive the sampling distribution from applying the Central Limit Theorem

Central Limit Theorem

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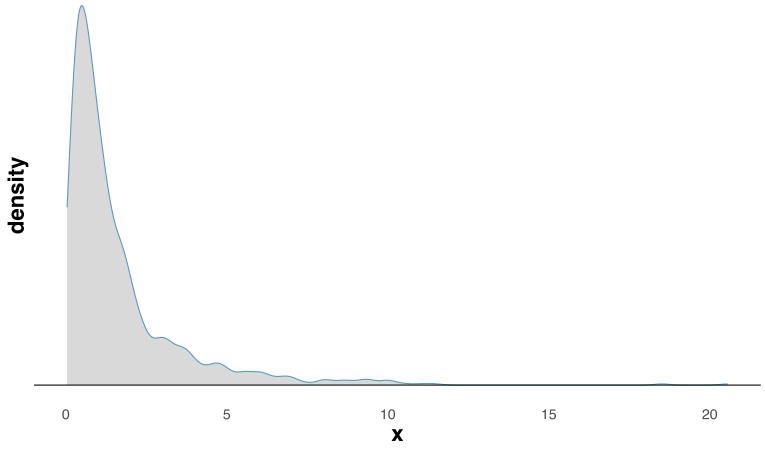
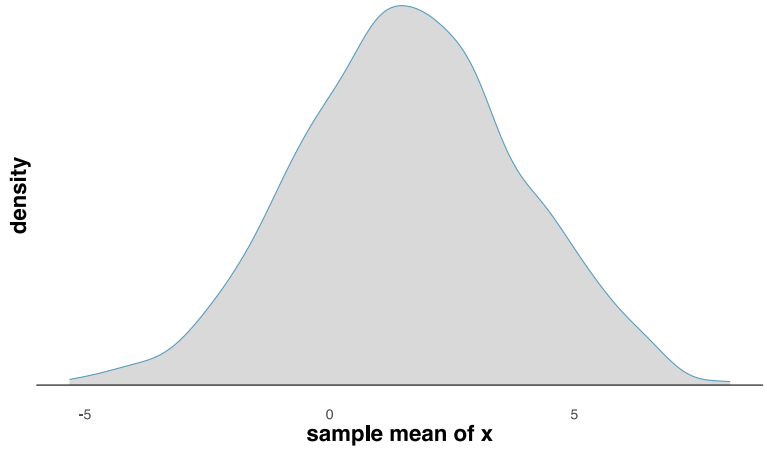
This is true *even if* the underlying data are not normally distributed!





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Question: How big does our sample need to be?

Answer: Rule of thumb is $n \geq 30$

But this is not a hard and fast rule! Be cautious about hypothesis testing and inference with small sample sizes.

Standard errors

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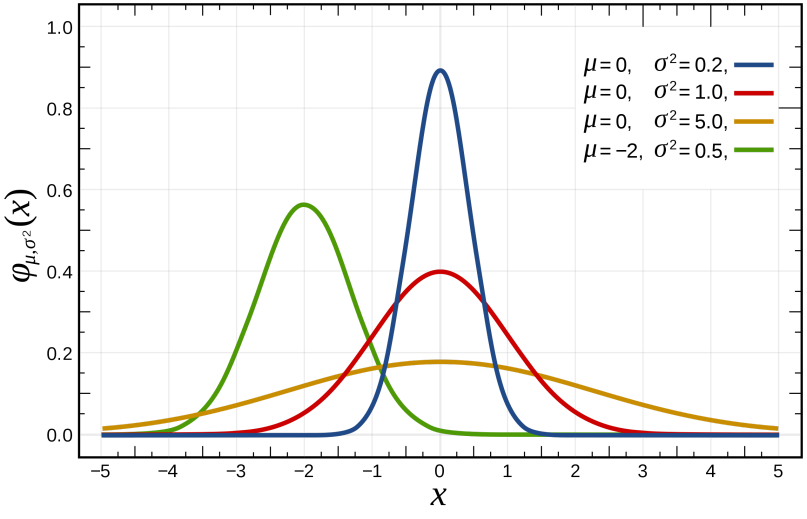
but there are so many normal distributions!

We need to know the μ and σ of our sampling distribution in order to fully model the variability of our statistic.



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  $\mu_{2023} - \mu_{2010} = 0$
- H_0: Male and female wages have a mean *difference* of zero.
  $\mu_{men} - \mu_{women} = 0$.
- H_0: There is *no effect* of neonicotinoid use on colony collapse disorder.
  $\beta_1 = 0$. (Note that linear regression parameters are conditional means -- mean of y conditional on a level of x)

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For example, if we are interested in a sample mean, our friend the **Central Limit Theorem** tells us that:

$$SE = \frac{s^2}{\sqrt{n}}$$

where s is the sample standard deviation and n is the sample size.

> Q: What happens to the standard error as sample size increases? Why?

Standard error for regression slope

The standard deviation of your sampling distribution is called the **standard error**

How you calculate the standard error depends on the research question.

For example, if we are interested in a regression slope, the CLT plus some algebra tell us that:

$$SE = \sqrt{var(\hat{\beta}_1)} = \sqrt{\frac{s^2}{\sum_i (x_i - \bar{x})^2}}$$

where s^2 is the sample variance of model errors $\hat{y}_i - y_i$

> Q: What happens to the standard error as sample size increases? Why?

SE for comparing two means

The standard deviation of your sampling distribution is called the **standard error**

How you calculate the standard error depends on the research question.

For example, if we are interested in the *difference* between two means, the CLT plus some algebra tell us that:

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

where s_k is the sample standard deviation in each of the two samples and n_k is the sample size in each sample

> In all these cases, the SE is the standard deviation of the sampling distribution!

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The **standard error** is our estimate of the the standard deviation of the sampling distribution. We can never actually collect hundreds of independent samples, so we use our single sample to approximate the true sampling distribution standard deviation, leveraging the **Central Limit Theorem**

Standard error measures how dispersed our sample statistic is around the population parameter of interest (highly dispersed = large SE = a lot of uncertainty about the population parameter from our one sample)

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Step 4: quantify the probability that your sample statistic differs from the null by the observed amount, if the null were true

- I know how that my sample statistic is drawn from a normal distribution with mean μ and an estimated standard deviation given by SE.

Hypothesis testing in five steps

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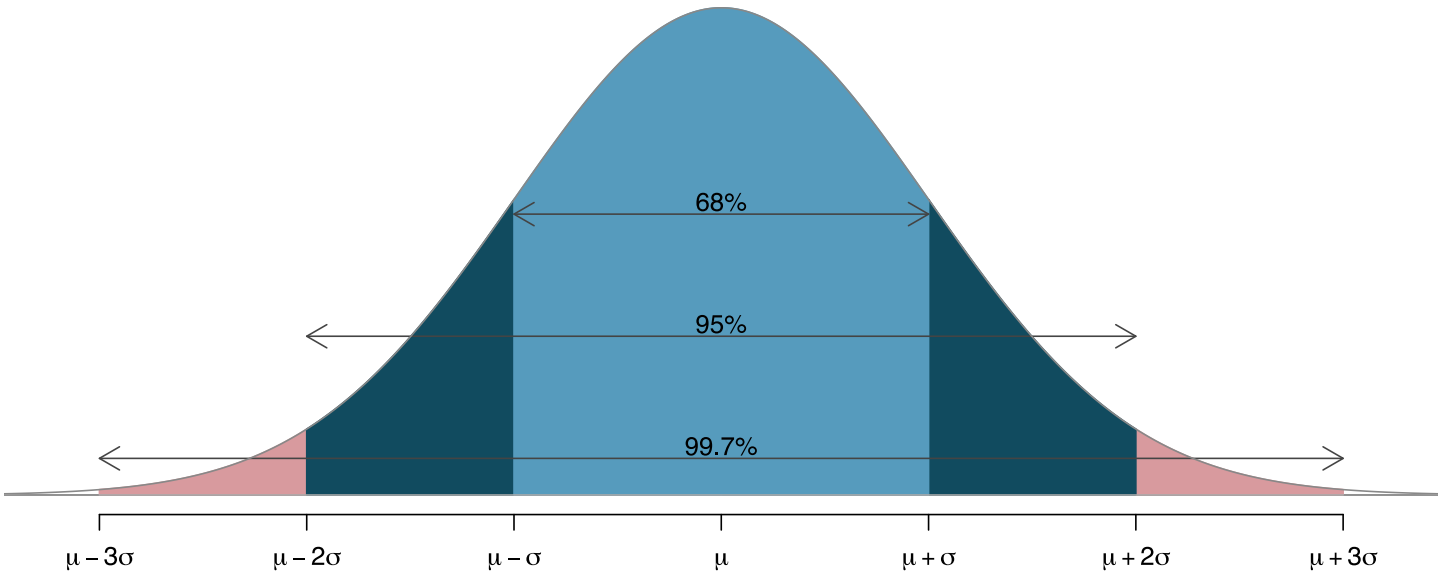
- Yep! But we need a couple more definitions to get all the way there.

The 68-95-99.7 rule

For a normal distribution:



Probabilities for falling within 1, 2, and 3 standard deviations of the mean in a normal distribution.

Z-score

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Z-score: How many standard deviations is a value from the mean?

$$z = \frac{x_i - \mu}{\sigma}$$

where μ is the mean and σ is the standard deviation

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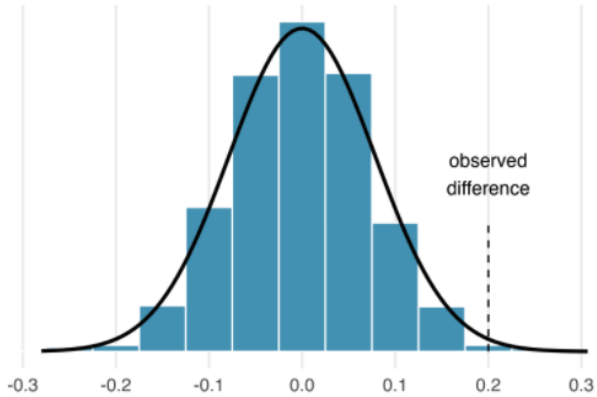

Quantifying probabilities: *p*-value

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> Question: What feature of the normal distribution lets us simplify this to $2 * Pr(Z > |z|)$?

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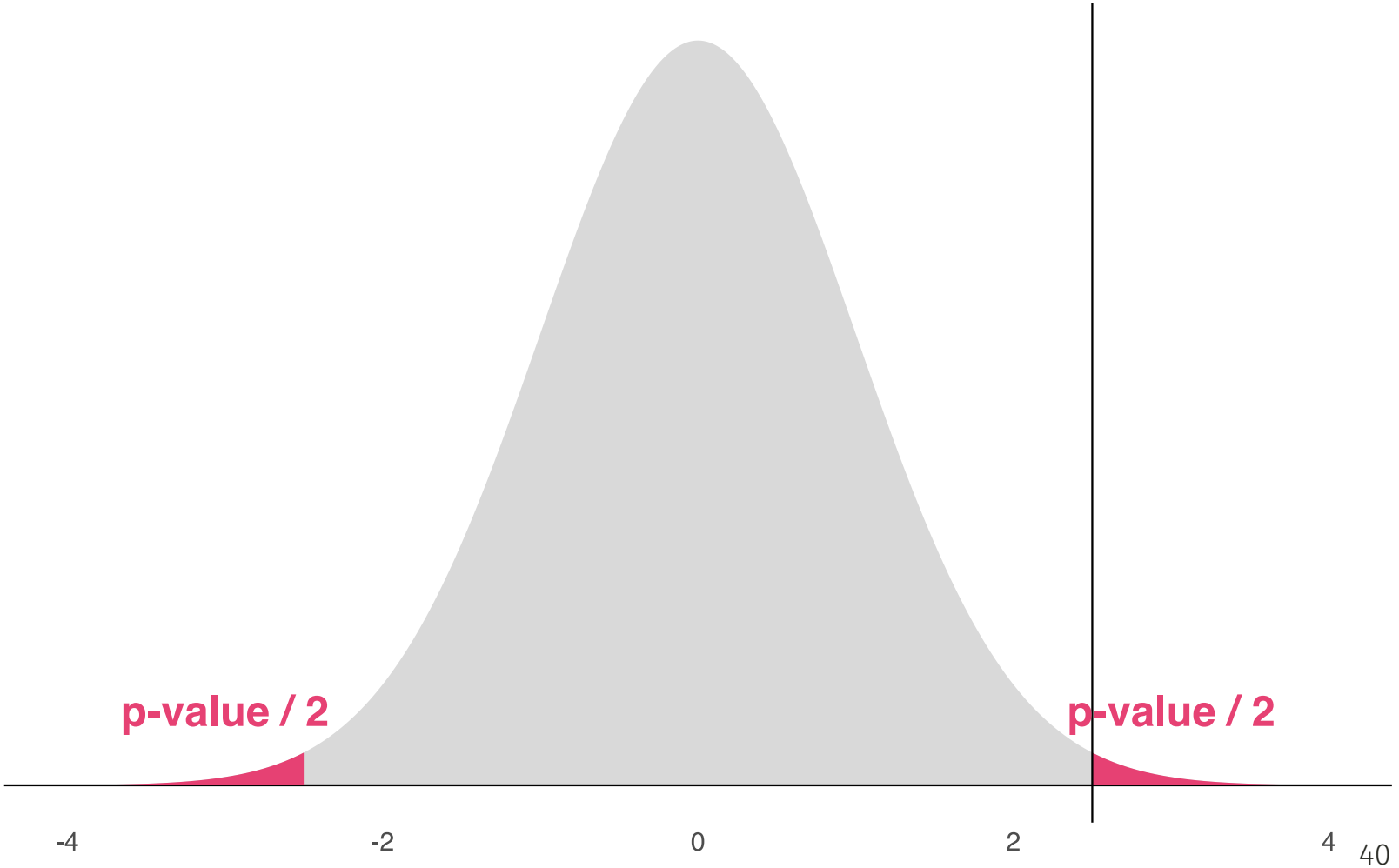

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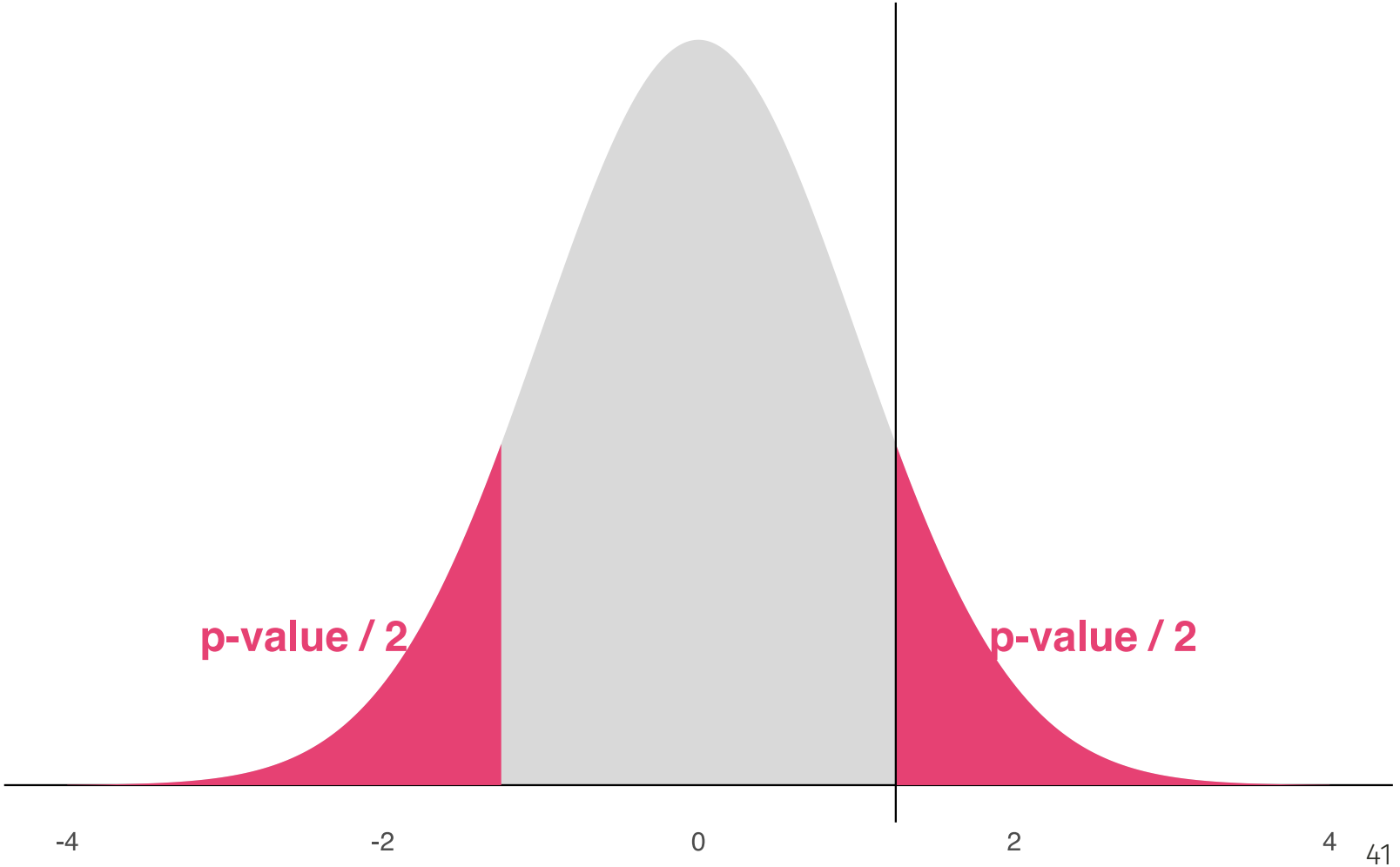

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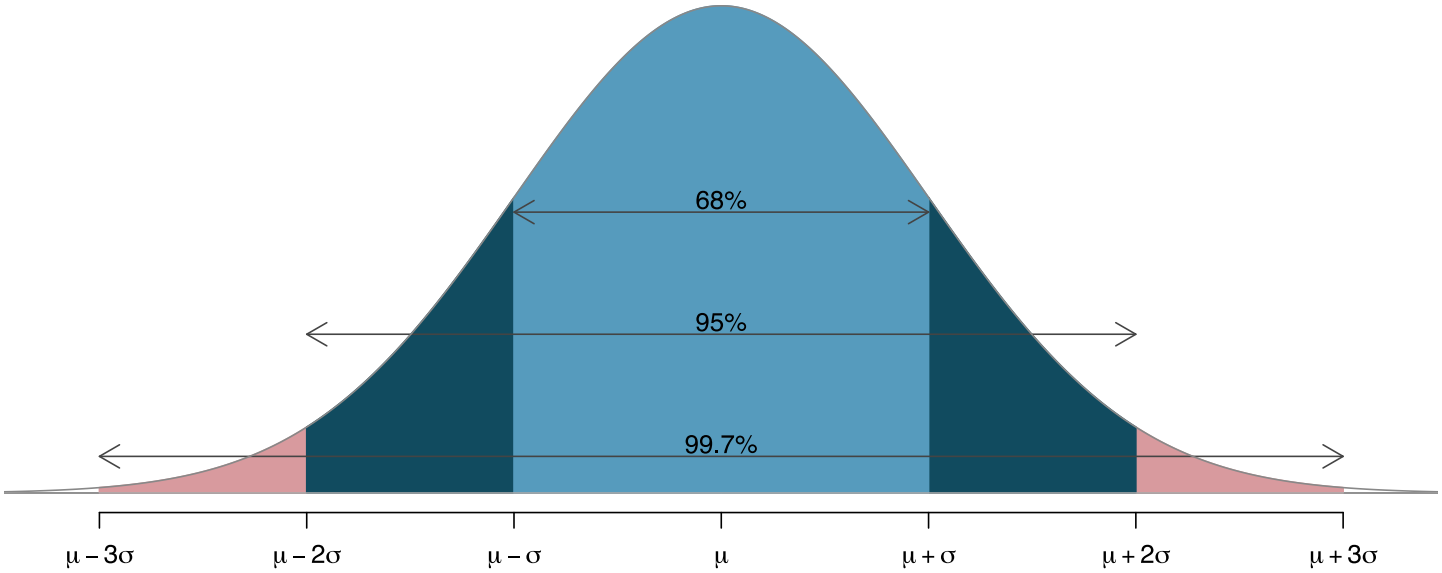

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How do I compute a *p*-value from a test statistic?

- **In math:** Integrate the sampling distribution's probability density function between $-\infty$ and $-|z|$; multiply by 2
- **In `R`:** `pnorm()`, `t.test()`, `summary(lm())`, ...

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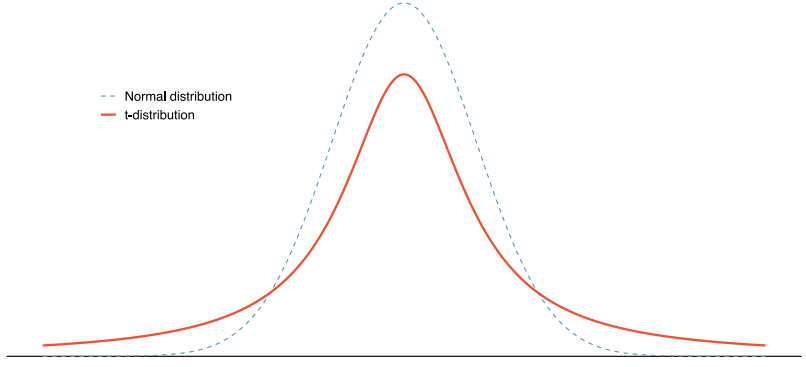
2. We have a **large enough sample** (at the very least $n \geq 30$)

When n is relatively small, we can still proceed, we just need to use a t-distribution (and T-score -- use `pt()` in `R`) instead of a normal distribution (and Z-score)



Hypothesis testing in five steps

The general framework for implementing a hypothesis test is:

1. **Define the null** and alternative hypotheses
2. Collect data and compute the **point estimate of the statistic**
3. Model the **variability of the statistic**
4. Given this variability, **quantify the probability that your sample statistic differs from the null** by the observed amount, if the null were true
5. Based on #4, either **reject** or **fail to reject** the null

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So...what is a low enough p-value to **reject the null**?

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- In general, reject the null if $p < \alpha$.
  - Other common αs: 0.01, 0.1

Statistical significance

We say a point estimate is "statistically significant" when:

$p < \alpha$

For example:

> "[W]e find a **statistically-significant** effect whereby increases in surface UV intensity lowers subsequent COVID-19 growth rates...we estimate that a 1 kJm−2hr−1 increase in local UV reduces local COVID-19 growth rates by .09 (±.04, p =.01) percentage points over the ensuing 17 days." --- *Carleton et al., 2021*

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- Failing to reject the null tells us we do not have sufficient evidence to prove there is an effect or a difference

Constructing confidence intervals

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- Effect sizes are important! We care not just about whether a treatment effects an outcome, but by *how much*

A **confidence interval** is a range of plausible values where we may find the true population value.

- It tells us something about the magnitude of the parameter of interest, as well as our uncertainty around our estimate

Confidence intervals

When the sampling distribution of a point estimate can be modeled as normal, the point estimate we observe will be within 1.96 standard errors of the true value of interest about 95% of the time (think back to the 68-95-99.7 rule).

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Thus, a 95% confidence interval for such a point estimate can be constructed:

$$\text{point estimate} \pm 1.96 * SE$$

We can be 95% confident this interval captures the true value.

> Also can see this as: `2*pnorm(-1.96) = .05`

Confidence intervals

You can build a confidence interval for any level of α:

$$\textbf{point estimate} \pm z_{\alpha/2} * SE$$

where $z_{\alpha/2}$ is a "critical value" that varies with significance level α.

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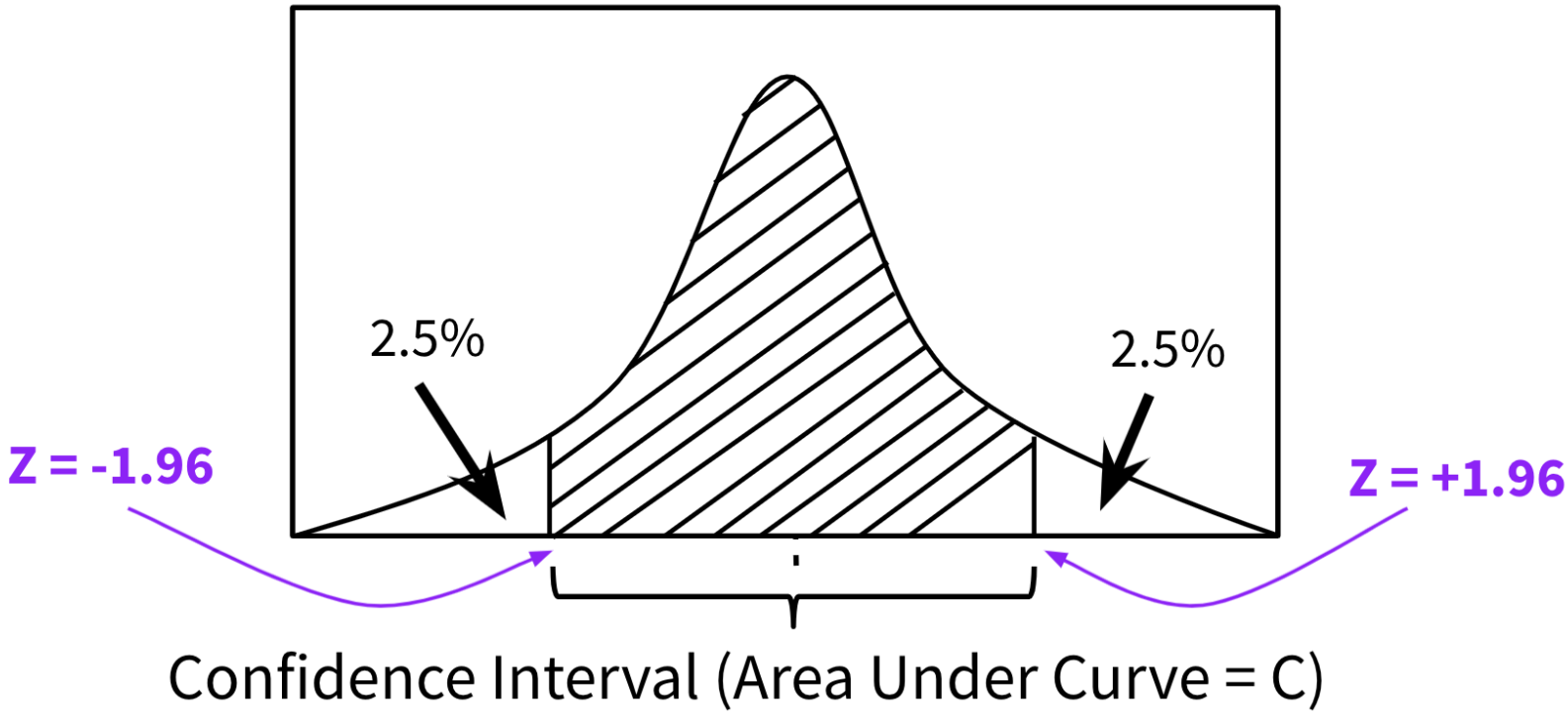
> Why do the z-scores get larger for higher confidence intervals?

A 95% confidence interval visual



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Why is this a big deal?

The population parameter *is not random*. So it either **is or is not** inside your CI.

Slides created via the R package **xaringan**.

Some slide components were borrowed from Ed Rubin's awesome course materials.